NEW SCHEME OF QUANTUM TELEPORTATION

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Abstract. A new scheme for quantum teleportation is presented, in which the complete teleportation can be occurred even when an entangled state between Alice and Bob is not maximal.

1. Introduction

The quantum teleportation has been discussed in several articles [2, 1, 4, 5, 3] by several different schemes. In most of models, complete (perfect) teleportation can be occurred if the entangled state between Alice and Bob is maximal. Here we reformulate the teleportation process and show in our model that the complete teleportation is possible even in the case for non-maximal entangled state.

2. Basic Setting

Let $\mathcal{H} = \mathbb{C}^n$ be a finite dimensional complex Hilbert space, in which the scalar product $< , >$ is defined as usual. Let e_n ($n = 1, \cdots, n$) be a fixed orthonormal basis (ONB) in $\mathcal{H}$, and let $B(\mathcal{H})$ be the set of all bounded linear operators on $\mathcal{H}$, which is simply denoted by M_n.In M_n, the scalar product (,) is defined by

$$(A, B) := trA^*B = \sum_{i=1}^{n} < Ae_i, Be_i >$$

Note that $e_{ij} := |e_i\rangle \langle e_j|$ $(i, j = 1, \cdots, n)$ is a ONB in M_n with respect to the above scalar product. The mappings

$$A \in M_n \to A^L := \sum\nolimits_{i=1}^{n} Ae_i \otimes e_i \in \mathcal{H} \otimes \mathcal{H},$$

$$A \in M_n \to A^R := \sum\nolimits_{i=1}^{n} e_i \otimes Ae_i \in \mathcal{H} \otimes \mathcal{H}$$

define the inner product isomorphisms from M_n into $\mathcal{H} \otimes \mathcal{H}$ such that

$$(A, B) =<< A^L, B^L >>=<< A^R, B^R >>,$$

where the inner products in $\mathcal{H} \otimes \mathcal{H}$ is denoted by $<< \cdot, \cdot >>$.

Let $L(M_n, M_n)$ be the vector space of all linear maps $\Phi : M_n \to M_n$. $M_n \otimes M_n$ is the set of all linear maps from $\mathcal{H} \otimes \mathcal{H}$ to $\mathcal{H} \otimes \mathcal{H}$. By analogy between M_n and $\mathcal{H} \otimes \mathcal{H}$, one can construct the inner product isomorphisms between $L(M_n, M_n)$ and $M_n \otimes M_n$ such as

$$\Phi \in L(M_n, M_n) \to \Phi^L := \sum\nolimits_{i,j=1}^{n} \Phi e_{ij} \otimes e_{ij} \in M_n \otimes M_n$$

$$\Phi \in L(M_n, M_n) \to \Phi^R := \sum\nolimits_{i,j=1}^{n} e_{ij} \otimes \Phi e_{ij} \in M_n \otimes M_n$$

The inner products in $L(M_n, M_n)$ is defined as follows:

$$((\Phi, \Psi)) := tr\Phi^* \Psi = \sum\nolimits_{i,j=1}^{n} (\Phi e_{ij}, \Psi e_{ij}).$$

One can easily verify that it is equal to

$$tr_{12}\Phi^{L*}\Psi^L = tr_{12}\Phi^{R*}\Psi^R,$$

where tr_{12} is the trace over the space $M_n \otimes M_n$, whose ONB is $\{e_{ij} \otimes e_{kl}\}$.

Let $\left\{f_\alpha; \alpha = 1, \cdots, n^2\right\}$ be another ONB in M_n so that one has $trf_\alpha^* f_\beta = \delta_{\alpha\beta}$. It is easy to check that the maps $\Phi_{\alpha\beta} \in L(M_n, M_n)$ defined by $\Phi_{\alpha\beta}(A) := f_\alpha A f_\beta^*$ for any $A \in M_n$ can be written as $\Phi_{\alpha\beta} = |f_\alpha)(f_\beta|$ and the set $\{\Phi_{\alpha\beta}\}$ is a ONB of $M_n \otimes M_n$. Moreover the corresponding elements $\Phi_{\alpha\beta}^L, \Phi_{\alpha\beta}^R \in M_n \otimes M_n$ form ONBs of $M_n \otimes M_n$.The explicit expression of $\Phi_{\alpha\beta}^L$ and $\Phi_{\alpha\beta}^R$ are

$$\Phi_{\alpha\beta}^L := \sum\nolimits_{i,j=1}^{n} f_\alpha e_{ij} f_\beta^* \otimes e_{ij} \text{ and } \Phi_{\alpha\beta}^R := \sum\nolimits_{i,j=1}^{n} e_{ij} \otimes f_\alpha e_{ij} f_\beta^*.$$

There exist some important consequences for the above isomorphisms:

(1) Any map $\Phi \in L(M_n, M_n)$ is uniquely written as

$$\Phi(A) = \sum c_{\alpha\beta}\Phi_{\alpha\beta}(A) = \sum c_{\alpha\beta} f_\alpha A f_\beta^* \text{ with some } c_{\alpha\beta} \in \mathbb{C}$$

(2) If $\Phi(A^*) = \Phi(A)^*$, then $c_{\alpha\beta} = \overline{c_{\alpha\beta}} \in \mathbb{R}$ and Φ^L, Φ^R are self-adjoint in $\mathcal{H} \otimes \mathcal{H}$.

(3) If $\Phi(A) = \Phi(A)^*$,that is, the matrix $C := (c_{\alpha\beta})$ is Hermitian, then Φ and Φ^L, Φ^R can be written in the following canonical forms:

$$\Phi(A) = \sum_{\alpha} c_\alpha g_\alpha A g_\alpha^*$$
$$\Phi^L = \sum_{\alpha,i,j} c_\alpha g_\alpha e_{ij} g_\alpha^* \otimes e_{ij}$$
$$\Phi^R = \sum_{\alpha,i,j} c_\alpha e_{ij} \otimes g_\alpha e_{ij} g_\alpha^*$$

where $\left\{g_\alpha; \alpha = 1, \cdots, n^2\right\}$ is some ONB in M_n and $c_\alpha \in \mathbb{R}$.

(4) From (3) it follows that for any ONB $\{f_\alpha\}$

$$P_\alpha := \Phi_{\alpha\alpha}^L = \sum\nolimits_{i,j=1}^{n} f_\alpha e_{ij} f_\alpha^* \otimes e_{ij},\ Q_\alpha := \Phi_{\alpha\alpha}^R = \sum\nolimits_{i,j=1}^{n} e_{ij} \otimes f_\alpha e_{ij} f_\alpha^*$$

are mutual orthogonal projections in $\mathcal{H} \otimes \mathcal{H}$ satisfying

$$\sum\nolimits_{\alpha=1}^{n^2} P_\alpha = \sum\nolimits_{\alpha=1}^{n^2} Q_\alpha = I \otimes I \ (I \text{ is unity of } M_n)$$

(5) A any state (density operator) σ_{12} on $\mathcal{H} \otimes \mathcal{H}$ can be written in the form

$$\sigma_{12} = \sum\nolimits_{\alpha=1}^{n^2} \lambda_\alpha Q_\alpha = \sum\nolimits_{\alpha=1}^{n^2} \lambda_\alpha \sum\nolimits_{i,j=1}^{n} e_{ij} \otimes f_\alpha e_{ij} f_\alpha^*$$

with $\sum_{\alpha=1}^{n^2} \lambda_\alpha = 1$ and $\lambda_\alpha \geq 0$. Put

$$\Theta(A) := \sum\nolimits_{\alpha=1}^{n^2} \lambda_\alpha f_\alpha A f_\alpha^*$$

for any $A \in M_n$. Then Θ is a completely positive (CP) map on M_n, and σ_{12} is written as

$$\sigma_{12} = \sum\nolimits_{i,j=1}^{n} e_{ij} \otimes \Theta(e_{ij}).$$

Let take $A \in M_n$ with $trA^*A = 1$, then A^L (A^R) is a normalized vector in $\mathcal{H} \otimes \mathcal{H}$ and it defines a state σ in $\mathcal{H} \otimes \mathcal{H}$ as $\sigma := |A^L >><< A^L|$.

Definition 1. *The above state σ is maximal entangled if $A^*A = AA^* = \frac{I}{n}$, equivalently, $A = \frac{1}{\sqrt{n}}U$ with some unitary operator U in $\mathcal{H}$.*

Remark 1. *One can construct an ONB $\left\{ f_\alpha = U_\alpha/\sqrt{n}; \alpha = 1, \cdots, n^2 \right\}$ with unitary U_α.Then the corresponding projections P and Q given above (4) are maximal entangled states.*

Definition 2. *The map $\Phi \in L(M_n, M_n)$ is said to be normalized if $\Phi(I) = I$, base preserving if $tr\Phi(A) = trA$ for all $A \in M_n$, selfadjoint if $\Phi(A)^* = \Phi(A^*)$ for all $A \in M_n$, positive if $\Phi(A^*A) \geq 0$ for all $A \in M_n$ and completely positive if $\sum_{i,j=1}^{n} < x_i, \Phi(A_i^* A_j) x_j > \geq 0$ for any x_i $(i = 1, \cdots, n) \in \mathcal{H}$ and any A_i $(i = 1, \cdots, n) \in M_n$. Note that the canonical form of completely positive map is given by Θ above.*

3. Usual Scheme of Quantum Teleportation

The quantum teleportation scheme is written as follows.

Step 0:: Alice has an unknown quantum state ρ on an N–dimensional subspace a Hilbert space $\mathcal{H}_1$ and she was asked to teleport it to Bob.

Step 1:: For this purpose, we need two other Hilbert spaces $\mathcal{H}_2$ and $\mathcal{H}_3$, $\mathcal{H}_2$ is attached to Alice and $\mathcal{H}_3$ is attached to Bob. Take an entangled state σ on $\mathcal{H}_2 \otimes \mathcal{H}_3$ having certain correlation between Alice and Bob and prepare an ensemble of the combined system in the state $\rho \otimes \sigma$ on $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$.

Step 2:: Alice performs a measurement of the observable $F := \sum z_\alpha P_\alpha$, involving only the $\mathcal{H}_1 \otimes \mathcal{H}_2$ part of the system in the state $\rho \otimes \sigma$. When Alice obtains z_α, according to the von Neumann (or Luder's) rule, after Alice's measurement, the state becomes

$$\rho_\alpha^{(123)} := \frac{(P_\alpha \otimes \mathbf{1})\rho \otimes \sigma(P_\alpha \otimes \mathbf{1})}{tr_{123}(P_\alpha \otimes \mathbf{1})\rho \otimes \sigma(P_\alpha \otimes \mathbf{1})}$$

where tr_{123} is the full trace on the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$.

Step 3:: Bob is informed which outcome was obtained by Alice. This information is transmitted from Alice to Bob without disturbance and by means of classical tools.

Step 4:: Having been informed an outcome of Alice's measurement, Bob performs a corresponding unitary operation (key) onto his system. That is, if the outcome was z_α, Bob operates a unitary operator W_α and change the state into

$$W_\alpha^* \left(tr_{12}\rho_\alpha^{(123)} \right) W_\alpha.$$

If this state is equal to the original state ρ which Alice sent, then the teleportation is succeeded.

Thus the problem of the quantum teleportation is that for any state ρ in $\mathcal{H}_1$ whether we can construct the entangled state between $\mathcal{H}_2$ and $\mathcal{H}_3$ and the key $\{W_\alpha\}$ such that $W_\alpha^* \left(tr_{12}\rho_\alpha^{(123)} \right) W_\alpha = \rho$.

In some models [2, 1, 4] complete teleportation is possible if the entangled state σ used for the teleportation and the projection P_α are maximally entangled.

4. New Scheme of Entanglement and Teleportation

We propose a new protocol for quantum teleportation. Let us take the conditions that all three Hilbert spaces $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ are $\mathbb{C}^n$.Let the state σ in $\mathcal{H}_2 \otimes \mathcal{H}_3 = \mathbb{C}^n \otimes \mathbb{C}^n$ be

$$\sigma = \sum\nolimits_{i,j=1}^{n} e_{ij} \otimes \Theta(e_{ij})$$

Here e_{ij}, Θ are those given in Section 2 with an ONB $\left\{ f_\alpha; \alpha = 1, \cdots, n^2 \right\}$ but are defined on $\mathcal{H}_2$ and $\mathcal{H}_3$. We set an observable F in $\mathcal{H}_1 \otimes \mathcal{H}_2$ to be measured by Alice as follows:

$$F = \sum_\alpha z_\alpha P_\alpha := \sum_\alpha z_\alpha \sum\nolimits_{i,j=1}^{n} g_\alpha e_{ij} g_\alpha^* \otimes e_{ij},$$

where $\left\{ g_\alpha; \alpha = 1, \cdots, n^2 \right\}$ is another ONB of M_n. Then we define the teleportation map for an input state ρ in $\mathcal{H}_1$ and the measured value z_α of Alice by

$$T_\alpha(\rho) := tr_{12}(P_\alpha \otimes \mathbf{1})\rho \otimes \sigma (P_\alpha \otimes \mathbf{1}).$$

Lemma 1. *The teleportation map T_α has the form $T_\alpha(\rho) = \Theta(g_\alpha \rho g_\alpha^*)$ for any ρ in $\mathcal{H}_1$.*

Proof. One can write $T_\alpha(\rho)$ as

$$\begin{aligned}
T_\alpha(\rho) &= \sum\nolimits_{i,j=1}^{n} \sum\nolimits_{k,l=1}^{n} \sum\nolimits_{t,s=1}^{n} tr\left(g_\alpha e_{ij} g_\alpha^* \rho g_\alpha e_{ts} g_\alpha^*\right) tr\left(e_{ij} e_{kl} e_{ts}\right) \Theta\left(e_{kl}\right) \\
&= \sum\nolimits_{i,j,s=1}^{n} tr\left(g_\alpha e_{ij} g_\alpha^* \rho g_\alpha e_{ti} g_\alpha^*\right) \Theta\left(e_{jt}\right) \\
&= \sum\nolimits_{i=1}^{n} < g_\alpha e_i, g_\alpha e_i > \sum\nolimits_{j,t=1}^{n} < e_j, g_\alpha^* \rho g_\alpha e_t > \Theta\left(e_{jt}\right) \\
&= \sum\nolimits_{j,s=1}^{n} tr\left(g_\alpha^* \rho g_\alpha e_{jt}\right) \Theta\left(e_{jt}\right) \\
&= \Theta\left(g_\alpha \rho g_\alpha^*\right)
\end{aligned}$$

□

It is easily seen that T_α is completely positive but not trace preserving. In order to consider the trace preserving map from T_α, let us consider the dual map $\widetilde{T}_\alpha$ of T_α, i.e., $trAT_\alpha(\rho) =: tr\widetilde{T_\alpha}(A)\rho$. Indeed it is

$$\widetilde{T}_\alpha(A) = g_\alpha^* \widetilde{\Theta}(A) g_\alpha, A \in M_n$$

where $\widetilde{\Theta}$ is the dual to Θ;

$$\widetilde{\Theta}(A) = \sum\nolimits_{\alpha=1}^{n^2} \lambda_\alpha f_\alpha^* A f_\alpha.$$

The map $\widetilde{T}_\alpha$ is normalizable iff $rank\widetilde{T}_\alpha(I) = n$, that is, the operator $\widetilde{T}_\alpha(I)$ is invertible. Put

$$\kappa_\alpha := \widetilde{T}_\alpha(I).$$

In this case the teleportation map $\widetilde{T}_\alpha$ is normalized as

$$\widetilde{\Upsilon}_\alpha := \kappa_\alpha^{-\frac{1}{2}} \widetilde{T}_\alpha \kappa_\alpha^{-\frac{1}{2}}.$$

The dual map Υ_α of $\widetilde{\Upsilon}_\alpha$ is trace preserving and it has the form as

$$\Upsilon_\alpha(\rho) = \Theta\left(g_\alpha \kappa_\alpha^{-\frac{1}{2}} \rho \kappa_\alpha^{-\frac{1}{2}} g_\alpha^*\right) = \sum\nolimits_{\beta=1}^{n^2} \lambda_\beta f_\beta g_\alpha \kappa_\alpha^{-\frac{1}{2}} \rho \kappa_\alpha^{-\frac{1}{2}} (f_\beta g_\alpha)^* .$$

It is important to note that this teleportation map is linear with respect to all initial states ρ.

Let us consider a special case of σ such that

$$\sigma = \sum\nolimits_{i,j=1}^{n} e_{ij} \otimes \Theta(e_{ij}) \text{ with } \Theta(\bullet) := f \bullet f^* \text{ and } trf^*f = 1.$$

That is, σ is a pure state. In this case, one has

$$T_\alpha(\rho) = (g_\alpha f)\,\rho\,(g_\alpha f)^*$$

and

$$\kappa_\alpha = (g_\alpha f)^*(g_\alpha f).$$

Remark 2. *If $g_\alpha = U_\alpha/\sqrt{n}$ and $f = V/\sqrt{n}$, where U_α and V are unitary operators, then $\kappa_\alpha = 1/n^2$,which corresponds to the usual discussion.*

Further, it follows that Υ_α is trace preserving iff $rank\,(g_\alpha) = rank\,(f) = n$, and in such a case one has

$$\Upsilon_\alpha(\rho) = (fg_\alpha)\,\kappa_\alpha^{-\frac{1}{2}} \rho \kappa_\alpha^{-\frac{1}{2}} (fg_\alpha)^*$$

Put

$$W_\alpha := fg_\alpha \kappa_\alpha^{-\frac{1}{2}},$$

which is easily seen to be unitary. Thus we proved the following theorem.

Theorem 1. *Given ONB $\{g_\alpha; \alpha = 1, \cdots, n^2\}$ and a vector f in M_n on the n-dimensional Hilbert space, if $rank\,(g_\alpha) = rank\,(f) = n$ is satisfied, then one can construct an entangled state σ and the set of keys $\{W_\alpha\}$ such that complete teleportation occurs.*

Note here that our teleportation protocol is not required that the entangled state is maximal for the complete teleportation. We will discuss an example of this point in the next section.

5. Example

Let us construct an example as mentioned in Sec.3. That is, we construct an entangled state given in the form: $\sigma = \sum_{i,j=1}^{n} e_{ij} \otimes \Theta(e_{ij})$ with $\Theta(\bullet) := f \bullet f^*$ and $trf^*f = 1$.Then it is possible in our protocol to teleport completely by means of non-maximal entangled state σ. The above state σ is pure, so that σ is maximally entangled iff $f = \frac{U}{\sqrt{n}}$ with some unitary operator u. Therefore if $rank\,(f) = n$ and $f \neq \frac{U}{\sqrt{n}}$, then σ is not maximally entangled.

We will consider a bit more general question: For a ONB $\{f_\alpha\}$ $(\alpha = 1, \cdots, n^2)$ in M_n,whether can we construct n^2 projections $Q_\alpha = \sum_{i,j=1}^{n} e_{ij} \otimes f_\alpha e_{ij} f_\alpha^*$ such that all Q_α are mutually orthogonal and not maximally entangled. This question is reduced to find out the basis $\{f_\alpha\}$ such that $rank\,(f_\alpha) = n$ for any α and $f_\alpha \neq \frac{U}{\sqrt{n}}$ with unitary U.

(1) A positive answer for the above question is given in the case $n = 2$, that is, M_2. Let S_α $(\alpha = 0, 1, 2, 3)$ are spin matrices;

$$S_0 = I, S_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, S_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, S_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and put

$$\omega_\alpha := \frac{S_\alpha}{\sqrt{2}} (\alpha = 0, 1, 2, 3).$$

Now we consider an orthogonal transformation $C : \mathbb{R}^4 \to \mathbb{R}^4$. In terms of $C := (C_{\alpha\beta})$ one defines a new basis $\{f_\alpha\}$ in M_2:

$$f_\alpha = \sum\nolimits_{\beta=0}^{3} C_{\alpha\beta}\omega_\beta. \tag{5.1}$$

Since $C_{\alpha\beta}$ is real and $\omega_\alpha = \omega_\alpha^*$, it implies that $f_\alpha = f_\alpha^*$ and the equality

$$\det f_\alpha = \frac{1}{2} \left(C_{\alpha 0}^2 - \sum\nolimits_{\beta=1}^{3} C_{\alpha\beta}^2 \right),$$

so that all f_α have rank 2 iff $\det f_\alpha \neq 0$. Such f_α $(\alpha = 0, 1, 2, 3)$ generate the corresponding projection $Q_\alpha = \sum_{i,j=1}^{n} e_{ij} \otimes f_\alpha e_{ij} f_\alpha^*$ on mutually orthogonal subspaces of $\mathbb{C}^2 \otimes \mathbb{C}^2$ such that Q_α $(\alpha = 0, 1, 2, 3)$ are non-maximal entangled states iff the transformation $\{\omega_\alpha\}$ to $\{f_\alpha\}$ can not be generated by unitary U such as $U\omega_\alpha U^* = f_\alpha$.

From the orthogonality relation to C, it follows that

$$C_{\alpha 0}^2 + \sum\nolimits_{\beta=1}^{3} C_{\alpha\beta}^2 = 1 \text{ and } \sum\nolimits_{\alpha=0}^{3} C_{\alpha 0}^2 = 1.$$

These relations tell us that $\det f_\alpha \neq 0$ iff $C_{\alpha 0}^2 \neq \frac{1}{2}$. Thus the relation $\sum_{\alpha=0}^{3} C_{\alpha 0}^2 = 1$ implies that $\det f_\alpha \neq 0$ iff $C_{\alpha 0}^2 > \frac{1}{2}$.

As an example, let us take the matrix C as the form

$$C := R_{01}(\theta_1) R_{02}(\theta_2) R_{03}(\theta_3),$$

where $R_{ab}(\theta)$ is the rotation in (a, b)-plan with angle θ.Then one finds

$$C = \begin{pmatrix} c_1 c_2 c_3 & -s_1 & -c_1 s_2 & -c_1 c_2 s_3 \\ s_1 c_2 c_3 & c_1 & -s_1 s_2 & -s_1 c_2 s_3 \\ s_2 c_3 & 0 & c_2 & -s_2 s_3 \\ s_3 & 0 & 0 & c_3 \end{pmatrix},$$

where $c_i := \cos\theta_i$ and $s_i := \sin\theta_i$. It is easy to check that f_α generate the projections Q_α, whose corresponding states are non-maximal entangled if $|s_3| > \frac{1}{2}$. This inequality can be realized by taking θ_3 properly, e.g., $\frac{\pi}{6} < \theta_3 < \frac{5\pi}{6}$.

(2) We can construct even simpler ONB $\{f_\alpha; \alpha = 0, 1, 2, 3\}$ generating non-maximal entangled states such as

$$f_0 = \begin{pmatrix} \cos\theta_1 & 0 \\ 0 & \sin\theta_1 \end{pmatrix}, f_1 = \begin{pmatrix} -\sin\theta_1 & 0 \\ 0 & \cos\theta_1 \end{pmatrix},$$
$$f_2 = \begin{pmatrix} 0 & \cos\theta_2 \\ \sin\theta_2 & 0 \end{pmatrix}, f_3 = \begin{pmatrix} 0 & -\sin\theta_2 \\ \cos\theta_2 & 0 \end{pmatrix}.$$

These are the rank=2 matrices for $0 < \theta_1,\ \theta_2 < \pi/2$, and they generate a non-maximal entangled state when $\theta_1,\ \theta_2 \neq \pi/4$.

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